Oxford Cambridge and RSA

Wednesday 24 June 2015 – Morning

A2 GCE MATHEMATICS

4731/01 Mechanics 4

QUESTION PAPER

* 4 4 2 1 9 8 8 4 0 7 *

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4731/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 A turntable is rotating at $3\,\text{rad}\,\text{s}^{-1}$. The turntable is then accelerated so that after 4 revolutions it is rotating at $12.4\,\text{rad}\,\text{s}^{-1}$. Assuming that the angular acceleration of the turntable is constant,

(i) find the angular acceleration, **[3]**

(ii) find the time taken to increase its angular speed from $3\,\text{rad}\,\text{s}^{-1}$ to $12.4\,\text{rad}\,\text{s}^{-1}$. **[2]**

2 The region bounded by the x-axis, the lines $x = 1$ and $x = 2$, and the curve $y = kx^2$, where k is a positive constant, is occupied by a uniform lamina.

(i) Find the exact x-coordinate of the centre of mass of the lamina. **[6]**

(ii) Given that the x- and y-coordinates of the centre of mass of the lamina are equal, find the exact value of k. **[4]**

3 Two planes, A and B, flying at the same altitude, are participating in an air show. Initially the planes are 400 m apart and plane B is on a bearing of 130° from plane A. Plane A is moving due south with a constant speed of $75\,\text{m}\,\text{s}^{-1}$. Plane B is moving at a constant speed of $40\,\text{m}\,\text{s}^{-1}$ and has set a course to get as close as possible to A.

(i) Find the bearing of the course set by B and the shortest distance between the two planes in the subsequent motion. **[5]**

(ii) Find the total distance travelled by A and B from the instant when they are initially 400 m apart to the point of their closest approach. **[6]**

4 **(i)** Write down the moment of inertia of a uniform circular disc of mass m and radius $2a$ about a diameter. **[1]**

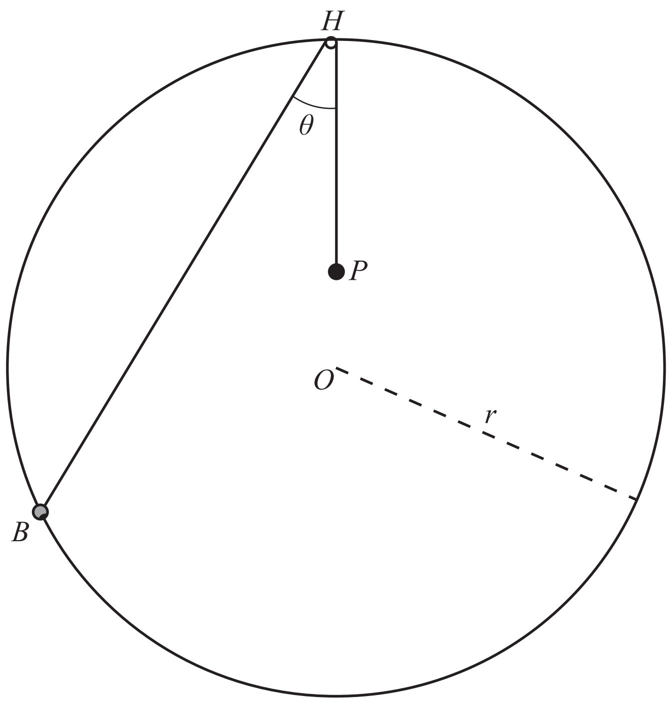
A uniform solid cylinder has mass M, radius $2r$ and height h.

(ii) Show by integration, and using the result from part **(i)**, that the moment of inertia of the cylinder about a diameter of an end face is

$$M\left(r^2 + \tfrac{1}{3}h^2\right)$$

and hence find the moment of inertia of the cylinder about a diameter through the centre of the cylinder. **[8]**

5



A smooth circular wire hoop, with centre O and radius r, is fixed in a vertical plane. The highest point on the wire is H. A small bead B of mass m is free to move along the wire. A light inextensible string of length a, where $a > 2r$, has one end attached to the bead. The other end of the string passes over a small smooth pulley at H and carries at its end a particle P of mass λm, where λ is a positive constant. The part of the string HP is vertical and the part of the string BH makes an angle θ radians with the downward vertical where $0 \leqslant \theta \leqslant \frac{1}{3}\pi$ (see diagram). You may assume that P remains above the lowest point of the wire.

(i) Taking H as the reference level for gravitational potential energy, show that the total potential energy V of the system is given by

$$V = mg(2\lambda r\cos\theta - 2r\cos^2\theta - \lambda a).$$ **[5]**

(ii) Find the set of possible values of λ so that there is more than one position of equilibrium. **[4]**

(iii) For the case $\lambda = \frac{3}{2}$, determine whether each equilibrium position is stable or unstable. **[6]**

6 A pendulum consists of a uniform rod AB of length $2a$ and mass $2m$ and a particle of mass m that is attached to the end B. The pendulum can rotate in a vertical plane about a smooth fixed horizontal axis passing through A.

(i) Show that the moment of inertia of this pendulum about the axis of rotation is $\frac{20}{3}ma^2$. **[3]**

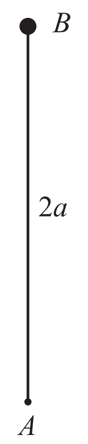


Fig. 1

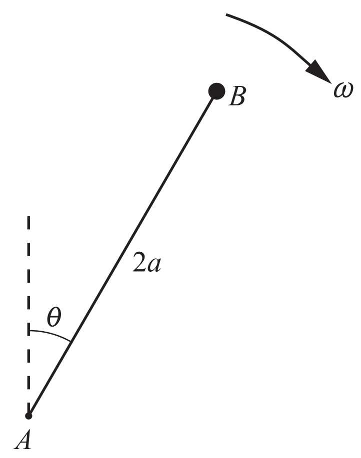


Fig. 2

The pendulum is initially held with B vertically above A (see Fig. 1) and it is slightly disturbed from this position. When the angle between the pendulum and the upward vertical is θ radians the pendulum has angular speed $\omega\ \text{rad s}^{-1}$ (see Fig. 2).

(ii) Show that

$$\omega^2 = \frac{6g}{5a}(1 - \cos\theta).$$ **[4]**

(iii) Find the angular acceleration of the pendulum in terms of g, a and θ. **[2]**

At an instant when $\theta = \frac{1}{3}\pi$, the force acting on the pendulum at A has magnitude F.

(iv) Find F in terms of m and g. **[7]**

It is given that $a = 0.735$ m.

(v) Show that the time taken for the pendulum to move from the position $\theta = \frac{1}{6}\pi$ to the position $\theta = \frac{1}{3}\pi$ is given by

$$k\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \operatorname{cosec}\left(\tfrac{1}{2}\theta\right)\mathrm{d}\theta,$$

stating the value of the constant k. Hence find the time taken for the pendulum to rotate between these two points. (You may quote an appropriate result given in the List of Formulae (MF1).) **[6]**

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